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Dependence of the induced loss factor on the coupling forms and coupling strengths: linear analysis

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Abstract

The influence of a set of satellite oscillators on the response behavior of a master oscillator, to which the set is coupled, is of fundamental significance to structural acoustics and beyond. The focus is largely on the induced loss factor that the satellite oscillators generate in the impedance of the master oscillator. Much of the research work performed on behalf of this investigation employed basically sprung masses for the satellite oscillators. A sprung mass is a primitive type of satellite oscillator and, as such, limitations are imposed on the range of applicability of these research works. In this paper more elaborate satellite oscillators are introduced; and, especially, a wider range of coupling forms and strengths are investigated. A number of new insights are, thereby, obtained. In particular, this paper facilitates further studies of the relationships among the linear impedance analysis, the energy analysis and the statistical energy analysis. These studies are in progress and will be subsequently reported.

1. Introduction

This paper investigates the influence exerted on a master oscillator by its coupling to a set of satellite oscillators [1]. (For those readers who may wish to consult a more extensive treatise on this subject matter, Ref. [1] is available upon request.) The master oscillator represents a master dynamic system and the set of satellite oscillators represents an adjunct dynamic system. The master dynamic system and the adjunct dynamic system constitute; in coupling, the complex (dynamic system) under investigation. The influence of concern is the loss factor that is added to the indigenous loss factor in the impedance that governs the motion of the master oscillator. This induced loss factor is acquired by the master oscillator by virtue of its coupling to the set of

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satellite oscillators. It was shown previously that if the satellite oscillators are merely masses (mass elements) and the coupling coefficients are merely springs (stiffness elements), the induced loss factor is independent of the loss factors that are assigned to the satellite oscillators [2–12]. This complex, comprising a master oscillator and a coupled set of sprung masses, is sketched in Fig. 1(a). The revelation that the induced loss factor is independent of the loss factors of the sprung masses caused several difficulties; none more stunning than "where did the energy go?" After all one may argue, this independence would permit vanishing values to be assigned to the loss factors of these sprung masses without affecting the induced loss factor! There were those who speculated that this independence may remain valid for situations in which the satellite oscillators and their couplings to the master oscillator may be different from those of sprung masses. Also, there were those who proposed that if one removes the restriction that the satellite oscillators are uncoupled to each other, a mechanism of damping may arise which will restore the balance of power and hence the conservation of energy [2–12]. These schemes were proposed despite the simplicity and the linearity of the equations of motion involved in the analysis. Notwithstanding that real dynamic systems may be modelled by a set of oscillators that are uncoupled to each other. Indeed,

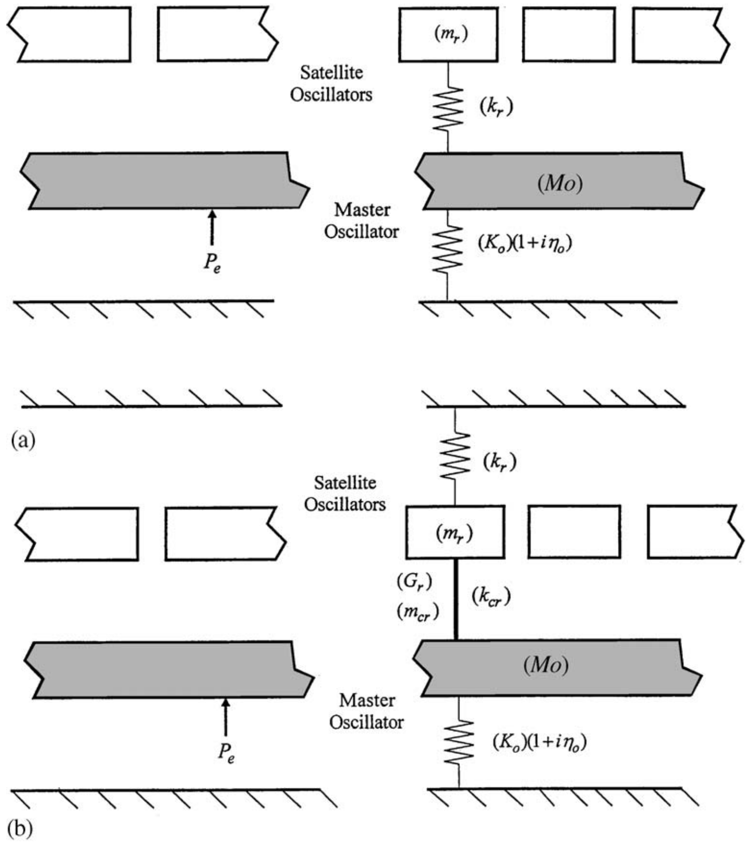

Fig. 1. (a) A master oscillator attached to a set of sprung masses; a sprung mass comprises a mass element coupled to the master oscillator via a stiffness element. (b) A set of satellite oscillators coupled to a master oscillator; the couplings comprise mass, stiffness and gyroscopic elements.

usually a resonating dynamic system may be analyzed in terms of modes that are by definition orthogonal. When the spatial dependence is removed by this orthogonality, the modes assume analytical forms commensurate with a set of oscillators that are uncoupled to each other. Does it then make sense that the mere lack of couplings among the satellite oscillators is the essential remedy for the violation of the law of conservation of energy?

In this paper not all the difficulties previously encountered are resolved; rather, minor, but significant extensions in scope are introduced. Although the restriction that the satellite oscillators are uncoupled to each other is retained, stiffness elements are introduced to the satellite oscillators so they may become true oscillators. Also, the couplings are no longer prescribed by stiffness elements alone, but, in addition, incorporate mass and gyroscopic elements as sketched in Fig. 1(b) [13, 14] (cf. Fig. 1(a)). This incorporation not only allows for variations in the coupling forms, but also in the coupling strengths. Under reasonable definitions the sprung masses indicate high coupling strength. In this way, moderate and weak coupling strengths may be integrated into the investigation. Imperatively, the independence of the induced loss factor from the values of the assigned loss factors of the satellite oscillators is properly interpreted and defined. In this definition, the law of conservation of energy remains intact without resorting to explanations transcending physics. Finally, preparations for an energy analysis (EA) of the elaborated complex are made. This energy analysis is to be subsequently reported [15].

Initially, the analytical developments are presented. The equations of motion are cast in terms of the impedances of the master oscillator, the satellite oscillators and the couplings. The couplings, again, are limited to those between the master oscillator and the individual satellite oscillators. Moreover, the external force drive is applied only to the master oscillator.

The resonance frequency distribution and the assigned loss factors of the satellite oscillators are specified. These loss factors are cast in terms of the associated modal overlap parameters. A number of simplifying assumptions are made with respect to defining parameters of the satellite oscillators. Under these assumptions estimates of the induced loss factor are made. Within the frequency span of the resonance frequencies and for values of modal overlap parameters less than unity, the induced loss factors, as functions of frequency, undulate. The excursions in these undulations increase as the modal overlap parameters decrease. Significantly, the mean-value levels of these induced loss factors coincide. This coincidence level is assumed by the induced loss factor when the modal overlap parameter is largely equal to unity. For values of the modal overlap parameters that exceed unity, the induced loss factors do not undulate and show higher erosions with increases in the modal overlap parameter. An erosion is defined as a decrease in level from that assumed by the mean-value level when the modal overlap parameter is largely equal to unity.

The expression for the induced loss factor, which comprise a summation over the individual satellite oscillators, is modified by extrapolations and interpolations so that the summation qualifies for replacement by integration. It is shown that such a replacement is tantamount to an averaging process that is reminiscent of the mean-value method proposed by Skudrzyk [16]. The first order approximation (FOA) of the integration yields an induced loss factor that assumes the level that this quantity attains when the summation is retained and the modal overlap parameter is largely unity. It is this FOA level of the induced loss factor that is independent of the modal overlap parameters and, by association, is independent of the loss factors assigned to the satellite oscillators.

Finally, the effects caused in the induced loss factor by statistical variations, in some of the parameters that define the satellite oscillators, are briefly considered. This summary is included in order to assess the impact that the simplification of parameters may have on the above results. It transpires that the variations in the parameters produce expected fluctuations in the levels of the induced loss factor. Graphical illustrations of these fluctuations are offered.

2. Analytical developments and definition of the complex to be analyzed

The impedance of the isolated master oscillator $Z_o^o(\omega)$ is defined by

$$Z_o^o(\omega) = (\mathrm{i}\omega M_o)\left[1 - (y)^{-2}\right]\left(1 = \mathrm{i}\eta_\mathrm{o}\right), \;\; y = \left(\omega/\omega_o\right), \;\; (\omega_o)^2 = \left(K_o/M_o\right), \tag{1a}$$

where (M_o) and (K_o) are the mass and stiffness elements of the master oscillator, respectively, and $\left(\eta_o\right)$ is the indigenous loss factor that is associated with the stiffness element; and $\left(\eta_o\right)$ is an assigned loss factor. Analogously, the impedance $Z_r(\omega)$ of the (r)th satellite oscillator in isolation is defined by

$$Z_r(\omega) = (\mathrm{i}\omega m_r)\left[1 - (z_r)^2\left(1 + \mathrm{i}\eta_r\right)\right], \; x_r = \left(\omega_r/\omega_o\right),$$
$$z_r = \left(x_r/y\right), \; (\omega_r)^2 = \left(k_{ro}/m_r\right), \tag{1b}$$

where (m_r) and (k_{ro}) are the mass and stiffness elements of the (r)th satellite oscillator, respectively, and $\left(\eta_r\right)$ is the indigenous loss factor that is associated with the stiffness element; again, $\left(\eta_r\right)$ is an assigned loss factor. It is further defined that the impedance $Z_{cr}(\omega)$ of the coupling between the master oscillator and the (r)th satellite oscillator be stated in the form

$$Z_{cr}(\omega) = (\mathrm{i}\omega m_r)\left[\bar{m}_{cr} - (z_{cr})^2\left(1 + \mathrm{i}\eta_{cr}\right)\right], \;\; \bar{m}_{cr} = (m_{cr}/m_{cr}),$$
$$x_{cr} = \left(\omega_{cr}/\omega_o\right), \;\; z_{cr} = \left(x_{cr}/y\right), \;\; (\omega_{cr})^2 = \left(k_{cro}/m_r\right), \tag{1c}$$

where (m_{cr}) and (k_{cro}) are the mass and the stiffness elements of the prescribed coupling, respectively, and $\left(\eta_{cr}\right)$ is the loss factor that is associated with the stiffness element; once again, $\left(\eta_{cr}\right)$ is an assigned loss factor. There is reason to define yet another impedance quantity $Z_{cr}^-(\omega)$ that is related to the coupling impedance $Z_{cr}(\omega)$, namely

$$Z_{cr}^-(\omega) = (\mathrm{i}\omega m_r)\left[\bar{m}_{cr} + (z_{cr})^2\left(1 + \mathrm{i}\eta_{cr}\right)\right]. \tag{1d}$$

To complete the definition of the coupling between the master oscillator and the (r)th satellite oscillator, a gyroscopic coupling coefficient (G_r) needs to be specified [13, 14]. It is convenient to normalize (G_r) in the form

$$g_r = \left[G_r/(\omega_o m_r)\right]. \tag{1e}$$

A sketch of the complex comprising the master oscillator coupled to a set of satellite oscillators is depicted in Fig. 1(b). In the present investigation the satellite oscillators comprise the adjunct dynamic system. It is assumed in this paper that the satellite oscillators are uncoupled to each other; the coupling is only between each satellite oscillator and the master oscillator and vice versa. The master oscillator comprises the master dynamic system. As already mentioned, much of the literature to date deals with a complex—a master dynamic system coupled to an adjunct dynamic system—in which a satellite oscillator is merely a mass element and the coupling of this oscillator to the master oscillator is merely a stiffness element; namely, a complex for which

$$k_{or} \cong 0, \quad m_{cr} \cong 0 \quad \text{and} \quad G_r \cong 0. \tag{2}$$

This specific complex is sketched in Fig. 1(a) [2–12]. Largely, the purpose in the present paper is to investigate the influence of negating the impositions stated in Eq. (2). The linear equation of motion of the master oscillator prior to its coupling to any set of satellite oscillators may be stated in the form

$$Z_o^o(\omega)V_o^o(\omega) = P_e(\omega), \tag{3}$$

where $V_o^o(\omega)$ is the response, $Z_o^o(\omega)$ is the impedance of the master oscillator in isolation, and $P_e(\omega)$ is the external force-drive to which the master oscillator is subjected. In this paper only the master oscillator is externally force driven and the superscript (o) designates quantities that pertain to the master oscillator in the absence of couplings—the master oscillator in isolation (cf. Eq. (1a)). On the other hand, in the presence of couplings and with the assistance of Fig. 1(b), the linear equations of motion of the master oscillator in situ and of a typical satellite oscillator in situ are derived. A straightforward algebraic manipulation of these equations yields

$$Z_o(\omega)V_o(\omega) = P_e(\omega), \quad V_r(\omega) = B_r(\omega)V_o(\omega), \tag{4}$$

where

$$Z_o(\omega) = Z_o^o(\omega) + \sum_1^R \left[\{Z_r(\omega)Z_{cr}(\omega)\} + (Q_{cr})^2\right][Z_r(\omega)Z_{cr}(\omega)]^{-1}, \tag{5a}$$

$$B_r(\omega) = \left[Z_{cr}^-(\omega) + G_r\right][Z_r(\omega) + Z_{cr}(\omega)]^{-1}, \tag{5b}$$

$$(Q_{cr})^2 = 4m_{cr}k_{cr} + (G_r)^2, \tag{5c}$$

the quantities $V_o(\omega)$ and $V_r(\omega)$ are the responses of the mass (M_o) of the master oscillator and the mass (m_r) of the (r)th satellite oscillator, respectively, (R) is the number of satellite oscillators that are coupled to the master oscillator, $P_e(\omega)$ is the external force drive that is applied externally to the master oscillator, (G_r) is the gyroscopic coupling coefficient and the quantities $Z_o^o(\omega)$, $Z_r(\omega)$, $Z_{cr}(\omega)$ and $Z_{cr}^-(\omega)$ are stated explicitly in Eq. (1) [13,14]. Again, straightforward manipulations and normalizations may cast Eq. (5) in the form

$$\begin{aligned} Z_o(\omega) &\Rightarrow Z_o(y) = (\mathrm{i}\omega M_o)\left[1 - (y)^{-2}\left\{[1 - S(y)] + \mathrm{i}\left[\eta_o + \eta_I(y)\right]\right\}\right], \\ B_r(\omega) &\Rightarrow B_r(y) = -\left[\bar{m}_{cr} + (Z_{cr})^2\left(1 + \mathrm{i}\eta_{cr}\right) - \mathrm{i}\left(g_r/y\right)\right], \end{aligned} \tag{6a}$$

where

$$(x_{rr})^2\left(1 + \mathrm{i}\eta_{rr}\right) = (x_r)^2\left(1 + \mathrm{i}\eta_{cr}\right)^2 + (x_{cr})^2\left(1 + \mathrm{i}\eta_{cr}\right), \tag{7a}$$

$$\begin{aligned} \left[S(y) - \mathrm{i}\eta_I(y)\right] &= \sum_1^R \left[S_r(y) - \mathrm{i}\eta_{Ir}(y)\right] \\ & \left[S_r(y) - \mathrm{i}\eta_{Ir}(y)\right] = (y)^2\left\{\bar{m}_r\left\{\left[1 - (z_r)^2\left(1 + \mathrm{i}\eta_r\right)\right]\right.\right. \\ & \left[\bar{m}_{cr} - (z_{cr})^2\left(1 + \mathrm{i}\eta_{cr}\right)\right] - \left(q_{cr}/y\right)^2\Big\} \\ & \left.\left[(1 + \bar{m}_{cr}) - (z_{rr})^2\left(1 + \mathrm{i}\eta_{cr}\right)\right]^{-1}\right\}, \end{aligned} \tag{7b}$$

and again

$$z_{rr} = \left(x_{rr}/y\right), \quad z_r = \left(x_r/y\right), \quad z_{cr} = \left(x_{cr}/y\right),$$
$$\left(q_{cr}/y\right)^2 = 4\bar{m}_{cr}(z_{cr})^2\left(1 + \mathrm{i}\eta_{cr}\right) + \left(g_r/y\right)^2, \ \ q_{cr} = \left[Q_{cr}/(\omega_o m_r)\right]. \tag{7c}$$

One notes that the compound coupling parameter (q_{cr}) is a function of the mass and gyroscopic coupling parameters $(\bar{m}_{cr})$ and (g_r), respectively. These coupling parameters are defined in Eqs. (1c) and (1e), respectively. One also notes, with satisfaction, that the dependence of the terms in the sum on the gyroscopic coupling parameter (g_r) is quadratic so that the sign assigned to the gyroscopic coefficient (G_r) plays no role in the influence of the individual satellite oscillators on the impedance of the master oscillator. The gyroscopic coupling is in quadrature to both the mass and the stiffness control couplings [13, 14].

Examination of Eqs. (6) and (7) shows that the normalized impedance that the satellite oscillators collectively induce on the master oscillator may be cast in terms of the two-vector $\left\{S(y), \ \ \eta_I(y)\right\} \ (R)$, which is dependent on (R), as indicated [1]. The evaluation of this two-vector, however, is predicated on explicitly specifying the two-vector $\left\{x_{rr}, \ \eta_{rr}\right\}$, its two supplemental components $\left\{x_r, \ \eta_r\right\}$ and $\left\{x_{cr}, \ \eta_{cr}\right\}$ and, finally, assigning the compound coupling (q_{cr}). The two-vector $\left\{x_{rr}, \ \eta_{rr}\right\}$ is designed, for the sake of convenience, to stay fixed with respect to variations in the index (r), $1 \leqslant r \leqslant R$. In this design, the springs that are placed on either side of the mass of a satellite oscillator are set to be similar. This similarity is expressed in the form

$$(x_r) = (\alpha_r)^{1/2}\left(x_r^o\right), \ \ (x_{cr}) = (\alpha_{cr})^{1/2}\left(x_r^o\right), \ \ (x_{rr}) = (\alpha_r + a_{cr})^{1/2}\left(x_r^o\right), \tag{8}$$

where (α_r) and (a_{cr}) are dubbed the spring factors. Further, by design, as in Ref. [12], here too (x_{rr}) is assigned a priori with equal numbers of resonance frequencies on either side of the resonance frequency (ω_o) of the master oscillator and the distribution is aligned in ascending order, namely

$$x_{rr} \leqslant, \ \ q = (r+1), \ \ 1 \leqslant r \leqslant (R-1). \tag{9}$$

Borrowing, again, from Ref. [12], $\left(x_r^o\right)$ is stated in the form

$$x_r^o = \left[1 + \{(1+R) - 2r\}\left(\gamma/2R\right)\right]^{-1/2}, \ \ \gamma < 1. \tag{10}$$

Examination of Eqs. (6) and (7) in the light of Eq. (8), shows that the normalized resonance frequencies of the satellite oscillators, in situ, are ascertained by satisfying the equality

$$(1 + \bar{m}_{cr}) = (z_{rr})^2, \ \ (1 + \bar{m}_{cr})(y)^2 = (x_{rr})^2, \ \ (x_{rr})^2 = (x_r)^2 + (x_{cr})^2,$$
$$(y)^2 = \left[(1 = m_{cr})^{-1}(\alpha_r + \alpha_{cr})\right]\left(x_r^o\right)^2. \tag{11a}$$

It is convenient to define the assigned loss factors $\left(\eta_r\right)$ and $\left(\eta_{cr}\right)$ of the satellite oscillators and the couplings in terms of the corresponding modal overlap parameters (b_r) and (b_{cr}), respectively. The definitions are in the forms

$$\eta_r = \left(b_r/y\right)[v_r(y)\omega_o]^{-1}, \ \ \eta_{cr} = \left(b_{cr}/y\right)[v_r(y)\omega_o]^{-1}.$$
$$\eta_{rr}(x_{rr})^2 = \eta_r(x_r)^2 + \eta_{cr}(x_{cr})^2 = \left[\left(\eta_r\alpha_r\right) + \left(\eta_r\alpha_{cr}\right)\right]\left(x_r^o\right)^2, \tag{12a}$$

where $v_r(y)$ is the modal density of the satellite oscillators and Eq. (8) is used [13]. Again, with the intended exceptions of the last section in this paper, it is convenient, without a great loss in generality, to assume that the spring factors (α_r) and (α_{cr}), the normalized coupling parameters (g_r) and $(\bar{m}_{cr})$ and the modal overlap parameters (b_r) and (b_{cr}) are to be independent of (r), namely

$$\alpha_r = \alpha, \quad \alpha_{cr} = \alpha_c, \quad g_r = g, \quad \bar{m}_{cr} = \bar{m}_c, \quad b_r = b_{cr} = b. \tag{13}$$

In addition and, again, as introduced in Ref. [12], the normalized mass $(\bar{m}_r)$ of the (r)th satellite oscillator is assumed to be independent of (r) and to be of the form

$$\bar{m} = \left(m/M_o\right) = \left(M_s/M_o\right)(R)^{-1}, \quad M_s = \sum_1^R (m_r). \tag{14}$$

Under the impositions stated in Eqs. (13) and (14), Eqs. (11a) and (12a) reduce to

$$y = x_r^o, \quad (1 + \bar{m}_c) = (\alpha + \alpha_c), \tag{11b}$$

$$\begin{gathered} \eta_r = \left(b/y\right)[v_r(y)\omega_o]^{-1}, \quad \eta_{cr} = \left(b/y\right)[v_r(y)\omega_o]^{-1}, \\ \eta_{rr} = \left(b/y\right)[v_r(y)\omega_o]^{-1}, \quad \eta_r = \eta_{cr} = \eta_{rr}, \end{gathered} \tag{12b}$$

respectively. One may argue that the modal density $v_r(y)$ of the satellite oscillators under the conditions stated in Eqs. (8) and (13) is given by

$$[v_r(y)\omega_o] = \left(2R/\gamma\right)\left(x_r^o\right)^{-3}, \quad y = \left(x_r^o\right). \tag{15}$$

The relationship between (y) and (x_r^o) stated in Eq. (11b) establishes, with the use of Eq. (10), a relationship between (y) and (r). Clearly, if (r) is discrete so is (y). Within the context of this paper (y) is treated, nonetheless, as a continuous variable. From Eqs. (12b) and (15) one obtains

$$\eta_r = \eta_{cr} = \eta_{rr} = b\left(\gamma/2R\right)\left(x_r^o\right), \quad \eta(y) = b\left(\gamma/2R\right)(y)^2. \tag{12c}$$

The resonance frequency distribution (x_r^o), as defined in Eqs. (10) and (11), is depicted, as a function of the index (r), in Fig. 2(a). In this figure the selected standard values of (R) and (γ) are used; these values are 27 and 0.6, respectively. In Fig. 2(b), $\left(\eta_r\right)$ is depicted, as a function of the index (r), for the standard values of (R) and (γ): $R = 27$ and $\gamma = 0.6$. In Fig. 2(b) three values of the modal overlap parameter (b) are employed; $b = 0.1$, 2.0 and 10. The standard value of (b) is selected to be 0.1. The discrete values of (x_r^o) and $\left(\eta_r\right)$ as functions of the index (r) are noted in Fig. 2; this discreteness is, by definition, expected.

Introducing the simplifications and impositions rendered in Eqs. (8–15) into Eq. (7), one may derive the explicit expression for the induced loss factor $\eta_I(y)$ in the form

$$\begin{aligned} \eta_I(y) &= \sum_1^R \eta_{Ir}(y), \\ \eta_{Ir}(y) &= -(y)^2 \mathrm{Im}\Big\{\bar{m}\Big\{\left[1 - \alpha\left(z_r^o\right)^2\{1 + \mathrm{i}\eta_r\}\right]\left[\bar{m}_c - \alpha_c\left(z_r^o\right)^2\{1 + \mathrm{i}\eta_r\}\right] \\ &\quad -4\bar{m}_c\alpha_c\left(z_r^o\right)^2\{1 + \mathrm{i}\eta_r\} - \left(g/y\right)^2\Big\} \\ &\quad \left[(1 + \bar{m}_c) - (\alpha + \alpha_c)\left(z_r^o\right)^2\{1 + \mathrm{i}\eta_r\}\right]^{-1}\Big\}, \quad z_r^o = \left(x_r^o/y\right), \end{aligned} \tag{16}$$

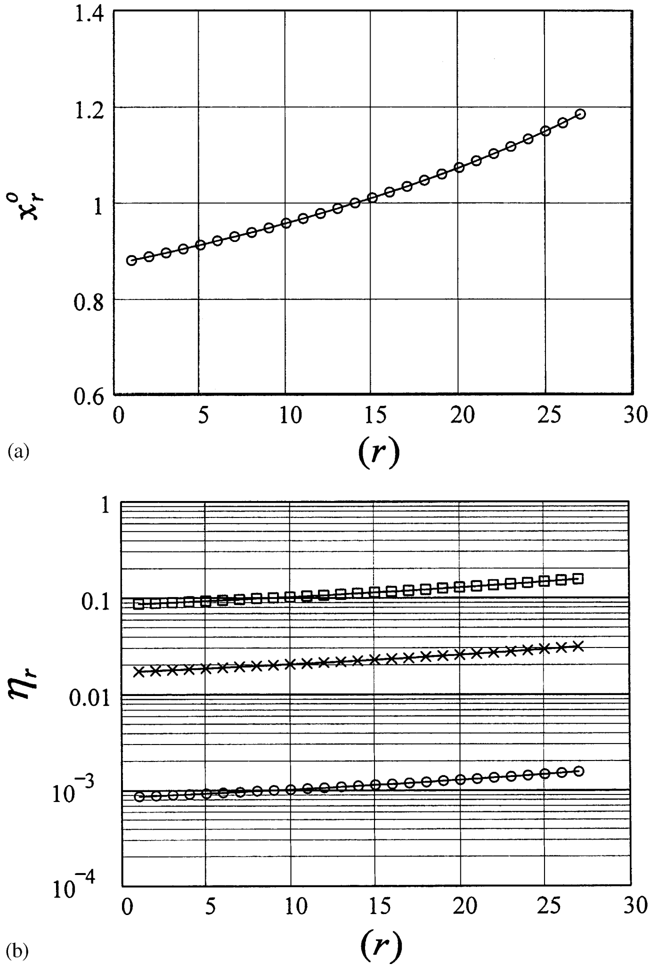

Fig. 2. (a) A resonance frequency distribution $x_{rr}(= x_r^o)$ of a set of satellite oscillators, as a function of the index (r) (cf. Eq. (10)): o, discrete (r); $-$, continuous (r). (b) The loss factor $\eta_{rr}(= \eta_r)$ assigned to a set of satellite oscillators as a function of the index (r) (cf. Eq. (12c)): o, modal overlap parameter (b) equal to (0.1); x, modal overlap parameter (b) equal to (2.0); $\square$, modal overlap parameter (b) equal to (10). o, x, $\square$, Discrete (r); $-$, continuous (r).

where $(\bar{m}_c)$, (g), (y), (α), (α_c), (x_r^o), $(\bar{m})$ and (η_r) are stated in Eqs. (1), (8), (10) and (12)–(14), respectively. The computations of $\eta_I(y)$ are largely carried out assigning the standard values

$$\bar{m}_c = g = 0, \quad \alpha_c = 1, \quad (M_s/M_o) = 10^{-1}, \quad b = (0.1)\,, \quad \gamma = 0.6 \ \text{ and } \ R = 27, \tag{17}$$

where $\bar{m}_c$, g and α_c define the couplings, (M_s) is stated in Eq. (14), (b) is the modal overlap parameter, (γ) is the frequency bandwidth parameter stated in Eq. (10), and (R) is the number of satellite oscillators in the set. When these standard assignments are deviated from, specific mentions are to be rendered, notwithstanding that, at times, the employment of these standard values may be reiterated. The computations performed on behalf of $\eta_I(y)$, as stated in Eq. (16), are presented graphically in Fig. 3 by the standard curve. Clearly, undulations in the induced loss factor $\eta_I(y)$, as a function of (y), are present in the levels of this quantity within the normalized frequency bandwidth $\Delta(y)$. This frequency bandwidth spans the resonance frequencies of the satellite oscillators; the frequency bandwidth, utilizing Eqs. (10) and (15), is given by

$$\Delta(y) \cong (\gamma/2), \qquad [1 + (\gamma/2)]^{-1/2} < y < [1 - (\gamma/2)]^{-1/2}. \tag{18}$$

Variations in the standard values in the computations are depicted in Figs. 3–5. In Fig. 3 the thinner curve and the thinnest curve represent the induced loss factor $\eta_I(y)$ with changes in the

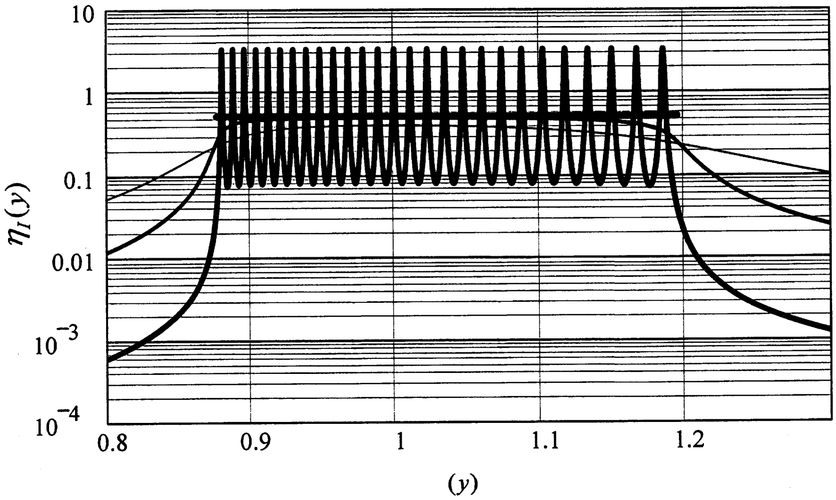

Fig. 3. Induced loss factor $\eta_I(y)$, as a function of (y), for a stiffness control coupling form with $\alpha_c = 1.0$ $[\alpha = 0.0]$, $g = \bar{m}_c = 0$ (sprung masses), and $R = 27$ and $(M_s, M_o) = 0.1$: $b = 0.1$; $b = 2.0$; $b = 10$; first order approximation (FOA).

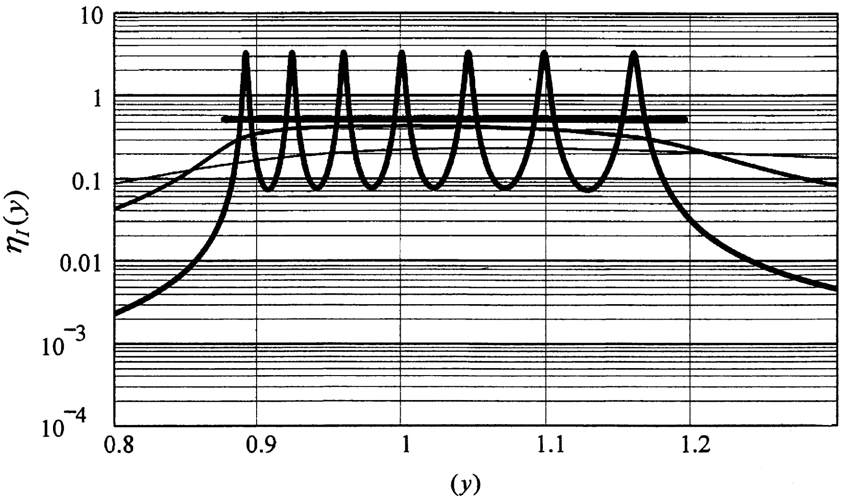

Fig. 4. Induced loss factor $\eta_I(y)$, as a function of (y), for a stiffness control coupling form with $\alpha_c = 1.0$ $[\alpha = 0.0]$, $g = \bar{m}_c = 0$ (sprung masses), and $R = 7$ and $(M_s, M_o) = 0.1$: $b = 0.1$; $b = 2.0$; $b = 10$; first order approximation (FOA).

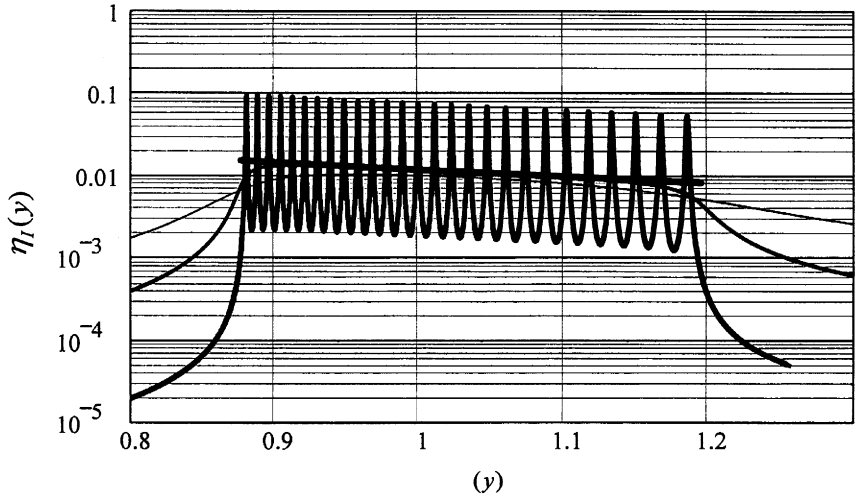

Fig. 5. Induced loss factor $\eta_I(y)$, as a function of (y), for a gyroscopically controlled coupling form $\alpha_c = \bar{m}_c = 0[\alpha = 1.0]$, $g = 0.15$ (moderate coupling), and $R = 27$ and $(M_s/M_o) = 0.1$: $b = 0.1$; $b = 2.0$; $b = 10$; first order approximation (FOA).

modal overlap parameter (b) from its standard value of 0.1–2.0 and then to 10.0, respectively. Since in both these changes (b) exceeds unity the undulations in $\eta_I(y)$, as a function of (y), are dramatically suppressed by these changes. Yet, some minor differences are observed in the levels of $\eta_I(y)$ pertaining to $b = 2.0$ and 10. Within the normalized frequency bandwidth defined in Eq. (18), the levels in the latter case are largely lower than those in the former. This decrease in levels is dubbed erosion. Fig. 3 is repeated in Fig. 4 except that the number (R) of satellite oscillators is changed from its standard value of 27–7. The differences between Figs. 3 and 4 are readily interpretable. Thus, since the couplings parameters $\bar{m}_c$, α_c and g and the mass ratio (M_s/M_o) remain the same, the levels and the undulations and their suppressions in Fig. 3 are duplicated in Fig. 4. On the other hand, in Fig. 5 the change is made only with respect to the coupling parameters; the change is from the standard values stated in Eq. (17) to $\bar{m}_c = \alpha_c = 0$, $g = 0.15$. This change is not only to weaker coupling, but to coupling that is governed by gyroscopic elements. One observes that except for much lower levels and slightly negative slopes in the curves toward higher values of (y), which is typical of gyroscopic coupling elements, Fig. 5 bears direct resemblance to Fig. 3 [1].

3. Replacing a summation by integration

The undulations that are present in the induced loss factor $\eta_I(y)$, as a function of (y), are a reflection of the modal character of the summand $\eta_{Ir}(y)$. The presence of this modal character is readily deciphered in Eq. (16). When this summand as a function of the index (r) becomes smooth so that its modal character is suppressed, $\eta_I(y)$, as a function of (y), is no longer undulated. This phenomenon is directly related to the modal overlap parameter (b) [13]. When (b) exceeds unity the modal character of the summand is suppressed. In this case, the summation qualifies to be replaced by an integration. In this integration the index (r) is assigned a continuous connotation; namely

$$\langle\eta_I(y)\rangle \geqslant \sum_1^R \eta_{Ir}(y) = \sum_1^R \eta_I(y,r)() \Rightarrow \int_{(1/2)}^{R+(1/2)} \eta_I(y,r)\,\mathrm{d}r, \quad b>1. \tag{19a}$$

A question arises: when (b) is less than unity, what can one do to suppress the modal character of the summand so that the summation can qualify to be replaced by integration? By extrapolations and interpolations one may introduce into the summand an artificial smoothness and, thereby, remove much of the modal character from this quantity. A manner of accomplishing this smoothness analytically is

$$\eta_I\left(y,R^1\right) \Rightarrow \sum_1^R \left(R^1\right)^{-1} \sum_{r^1=\left(-R^1/2\right)}^{\left(R^1/2\right)} \eta_I\left[y, r+\left(r^1/R^1\right)\right], \quad b<1, \tag{19b}$$

where (R^1) is an integer that is chosen large enough to attain the smoothness required of the summand so that the replacement of the summation by integration qualifies. If the summation in Eq. (19b) is absolutely (non-conditionally) convergent then from Eq. (19b) one may derive

$$\langle\eta_I(y)\rangle \Rightarrow \int_{(1/2)}^{R+(1/2)} \eta_I(y,r)\;\mathrm{d}r, \quad b<1, \tag{19c}$$

where (r) assumes here a continuous form. It is clear from Eq. (19b) that the level of $\langle\eta_I(y)\rangle$ is an averaged level of the induced loss factor $\eta_I(y)$ stated in Eq. (16). It transpires that this averaging procedure is commensurate with the mean-value method proposed by Skudrzyk [16]. From Eqs. (19a) and (19c) one obtains

$$\langle\eta_I(y)\rangle \Rightarrow \int_{(1/2)}^{R+(1/2)} \eta_I(y,r)\;\mathrm{d}r, \tag{20}$$

where the reference to the value of (b) becomes superfluous. Casting $\eta_{Ir}(y)$ of Eq. (16) in the form $\eta_I(y,r)$ and placing that quantity in Eq. (20) the integral is ready to be performed. The performance is facilitated by rendering a suitable transformation of the variable over which the integral needs to be carried out. With such transformation of variable one readily derives

$$\langle\eta_I(y)\rangle = D\left[C(y) + O\{\eta(y)\}^2\right], \quad \langle\eta_I(y)\rangle \Rightarrow \eta_I^1(y) = DC(y) \tag{21}$$

with

$$D = \left[(\pi/2)(2R/\gamma)(M_s/M_o)\right], \tag{22a}$$

$$C(y) = \left[(\bar{m}_c+\alpha_c)^2 + (g/y)^2\right][1+\bar{m}_c]^{-1}, \quad (1+\bar{m}_c) = (\alpha+\alpha_c), \tag{22b}$$

$$O = [(1+\bar{m}_c-\alpha_c)\alpha_c], \tag{22c}$$

where $\eta_I^1(y)$ is the FOA to $\langle\eta_I(y)\rangle$ and the validity of Eq. (21) is restricted to the frequency bandwidth defined in Eq. (18) [17]. The quantity $C(y)$ is dubbed the coupling factor. The coupling factor may be employed to categorize the strength of the coupling. The categorization may be expressed in the form

$$\geqslant 1, \text{ strong coupling,} \quad (23a)$$

$$C(y) \;\cong 3\times 1^{-2}, \text{ moderate coupling,} \quad (23b)$$

$$\leqslant 10^{-3}, \text{weak coupling} \quad (23c)$$

The term $O\{\eta(y)\}^2$, in Eq. (21), is of the order of the higher approximations to the integral, notwithstanding that situations exist in which (O) is identically zero; e.g., when the satellite oscillators are sprung masses for which $\alpha_c = 1, \quad g = \bar{m}_c = 0$. In passing it is observed that for the sprung masses the coupling factor is unity which categorizes sprung masses as strongly coupled. Significantly, $\eta_I^1(y)$ and, therefore, the FOA of $\langle \eta_I^1(y)\rangle$ is independent of the modal overlap parameter (b). Largely, it is this independence that caused some of the difficulties mentioned earlier in the introduction. These difficulties stem from the notion that the modal overlap parameter (b), and consequently the indigenous loss factors of the satellite oscillators $\eta(y)$, may be assigned, a priori, the value zero [2–10]. Eq. (19) makes clear that the validation of $\langle \eta_I^1(y)\rangle$ is called into question by this assignment. The FOA $\eta_I^1(y)$ of the mean-value $\langle \eta_I(y)\rangle$ of the induced loss factor, as a function of (y), is depicted graphically in Figs. 3–5 by the thicker curve. The impositions on $\eta_I^1(y)$ correspond to those imposed on $\eta_I(y)$ depicted in these same figures. A remarkable property of the FOA of $\langle \eta_I(y)\rangle$, namely $\langle \eta_I^1(y)\rangle$, emerges when this quantity is superimposed on the respective Figs. 3–5. It is now observed, in these figures, that the mean-value levels of the undulations in $\eta_I(y)$, when (b) is small compared with unity, $b \ll 1$, converges onto the levels of the FOA of this quantity, namely it converges on $\eta_I^1(y)$ [12,15]. Conversely, when (b) is small compared with unity, exhibiting $\eta_I^1(y)$ alone and neglecting to mention that these mean-value levels are substituted for highly undulated levels, is not a viable scientific procedure, unless ignorance is bliss [12]. On the other hand, when (b) approaches and exceeds unity, the levels of $\eta_I(y)$ become free of undulations, but these levels erode with further increases of (b). The erosion is revealed in Figs. 3–5 by comparing $\eta_I(y)$ with $\eta_I^1(y)$ in these figures, once again, remembering that the latter quantity is independent of (b). The erosion worsens as the modal overlap parameter (b) reaches higher and higher above unity; e.g., compare the curves, in Figs. 3–5, pertaining to $b = 2.0$ with the corresponding curves pertaining to $b = 10$ [1].

4. A typical member of an ensemble of complexes supporting various parametric combinations

In the preceding evaluations the distribution of resonance frequencies (x_{rr}) and the assigned loss factors (η_{rr}) for the satellite oscillators are sequential functions of the index (r). These two quantities, which are stated in Eqs. (11) and (12) and exemplified in Figs. 2(a) and 2(b), respectively, may be smoothed out by extrapolations and interpolations into monotonic and continuous functions of a continuous (r). This kind of smoothness is rarely found in practice and a question arises as to what are the expected consequences of more practical assignments for these parameters and others? In this section a few layers are removed in the quest to discover the phenomena that may be encountered in the induced loss factor $\eta_I(y)$ by the insertion of these more realistic parametric values. Since the assignments for the parameters that define the satellite oscillators and their couplings to the master oscillator, can hardly be drawn, a more generalized approach is undertaken to investigate the influence of introducing variations in these parametric values. In particular, in this section two parameters are selected to carry these variations; either individually or in unison. In the first, the index (r) of a satellite oscillator is assigned a pseudo-statistical value. (Pseudo-statistical is in reference to a sample selected out of an ensemble of random samples.) The index (r) is distributed sequentially and fractionally, in the range $1 \leqslant r \leqslant 27$. A pseudo-statistical index is designated $\Lambda(r)$, where $\Lambda(r)\hat{o}\Lambda(q)$, $q = (r+1)$, $1 \leqslant r \leqslant (R-1)$ (cf. Eq. (9)). In the second, the modal overlap parameter (b_r), with $b_r = b_{cr}$, is assigned a pseudo-statistical value that is distributed in the ranges $(2) \geqslant b_r \geqslant (0.1)$ and $(3.5) \geqslant b_r \geqslant (1)$, respectively.

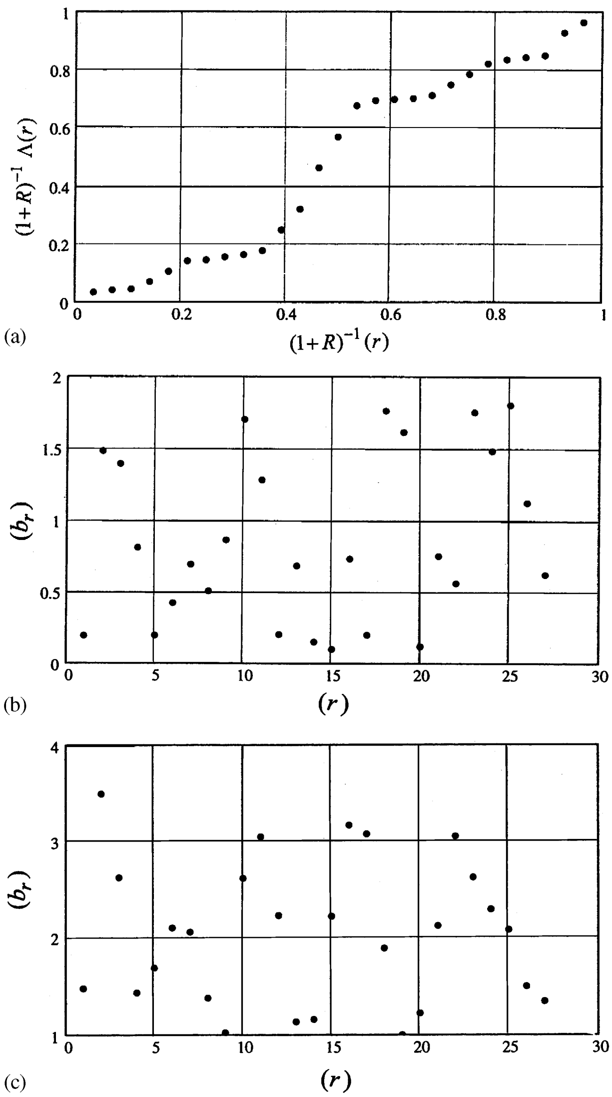

Fig. 6. Pseudo-statistical variations: (a) in the normalized index $\Lambda(r)(1+R)^{-1}$, as a function of the normalized (integer) index $(r)(1+R)^{-1}$; (b) in the modal overlap parameter (b_r); $(0.1) \leqslant b_r \leqslant (2.0)$; (c) in the modal overlap parameter (b_r); $(1.0) \leqslant b_r \leqslant (3.5)$.

The distribution of $\Lambda(r)$ and of (b_r), with $R = 27$, that are employed in this section are depicted graphically in Figs. 6(a)–(c). The two-vector $\{(x_{rr}),\ (\eta_{rr})\}$, as stated in Eqs. (8) and (10–12), is typically depicted, for the pseudo-statistical values shown in Figs. 6(a),(b) and (c), in Figs. 7(a), (b)

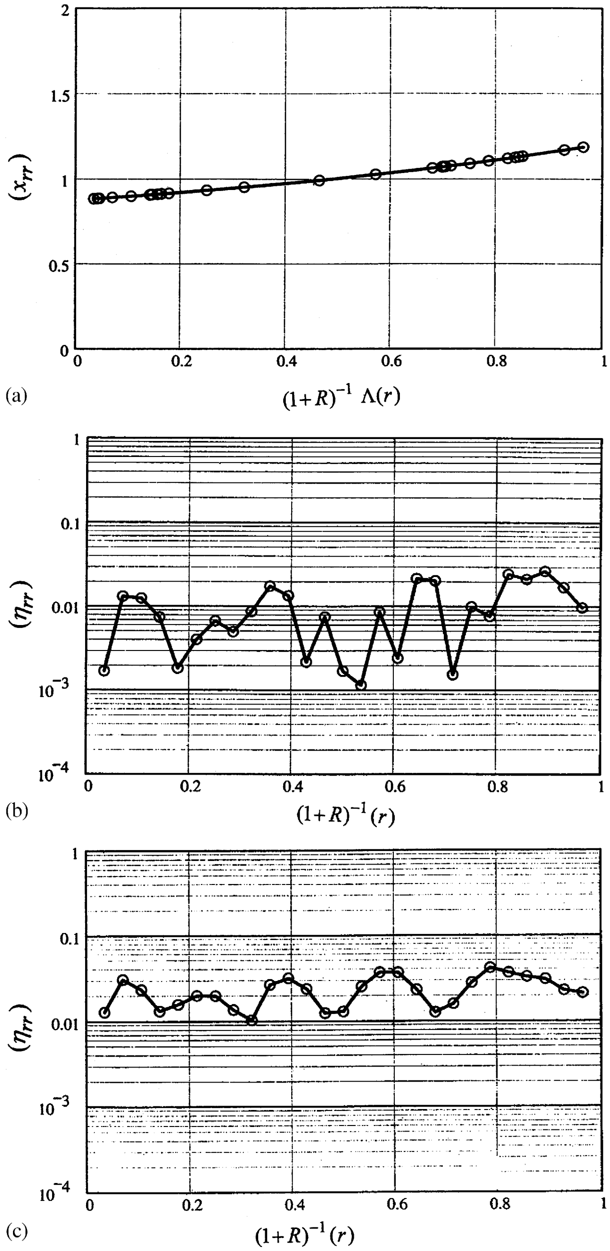

Fig. 7. Modification of pseudo-statistical variations: (a) in the resonance frequency distribution of satellite oscillators, as a function $\Lambda(r)$, with $\Lambda(r)$ depicted in Fig. 6(a); (b) in the loss factors assigned to individual satellite oscillators with (b_r) as depicted in Fig. 6(b); (c) in the loss factors assigned to individual satellite oscillators, with (b_r) as depicted in Fig. 6(c).

and (c), respectively. Fig. 7(a) depicts (x_{rr}) as a function of $\Lambda(r)$ and Figs. 7(b) and (c) depict (η_{rr}), as a function of (r). (cf. Fig. 2). It is observed, in Fig. 7(a), that the pseudo-statistical variations embody the phenomenon of mode bunching in which variations in the modal density of the satellite oscillators drastically vary as a function of $\Lambda(r)$ [18]. On the other hand, as Figs. 7(b) and (c) show, the loss factor (η_{rr}), as a function of (r), faithfully follows the variations assigned to (b_r). In Fig. 7(b) some of the values of (b_r) are less than unity, in Fig. 7c all the values of (b_r) are in excess of unity.

The influence of the variations, described in Figs. 6 and 7, on the induced loss factor $\eta_I(y)$, as a function of (y), is exemplified in Fig. 8. This figure represents a set of figures. This set of figures presents a complete evolution in the process of applying the pseudo-statistical variations depicted in Figs. 6 and 7 to the two parameters $\Lambda(r)$ and (b_r). The coupling in Fig. 8 is strong and is defined by $\alpha_c = 1.0[\alpha = 0.0]$, $\bar{m}_c = g = 0$, which pertains to sprung masses for the satellite oscillators (cf. Eq. (23)). Corresponding treatments for moderate and weak couplings exhibit, except for the levels of $\eta_I(y)$, the same results; e.g., cf. Figs. 8 and 9 [1]. In Fig. 9 the coupling is moderate; $C(y) \cong 0.022$; see Eq. (23). The first figure in the set, e.g., Fig. 8a, as well as Fig. 9, depict the base situation in which $\Lambda(r) = r$ and $b_r = 1$. Both, Figs. 8(a) and 9, exhibit undulations in the levels of the induced loss factor $\eta_I(y)$, as a function of (y). However, the excursions of these undulations are small and they are completely suppressed as soon as (b_r) approaches the value of (2) (cf. Figs 3–5). Moreover, it is noted that there is a tinge of edge erosion in Fig. 8a and even in Fig. 9. To confirm this statement and to provide for convenient and interpretable data from which to judge the more erratic data that incorporate the pseudo-statistical variations, the FOA of $\eta_I(y)$, namely $\eta_I^1(y)$, stated in Eq. (21), is superimposed on Figs. 8(a) and 9, and indeed on all other figures in the series entitled Fig. 8. The quantity $\eta_I^1(y)$, in Figs. 8 and 9, is depicted by the bold curves. (cf. Figs. 3–5). The second figure in the set, e.g., Fig. 8(b), depicts the situation in which $\Lambda(r)$ is as shown in Figs. 6a and 7a and $b_r = 1$. From Fig. 8(b), it is observed that at a mode bunching (a rich modal density) region the influence of the satellite oscillators, as expressed by $\eta_I(y)$, is more pronounced than at a mode sparsity (a poor modal density) region [18]. The third figure in the set, e.g. Fig. 8(c), depicts the situation in which $\Lambda(r) = r$ and (b_r) is as shown in Figs. 6(b) and 7(b); i.e., the pseudo-statistical variations in the modal overlap parameter (b_r) entertains values that are small compared with unity, and, therefore, as Fig. 8(c) shows, the levels of $\eta_I(y)$, as a function of (y), tend to fluctuate. These fluctuations are most pronounced at and in the vicinity of the resonance frequencies of those satellite oscillators to which the small values of (b_r) are assigned. The fourth figure in the set, e.g. Fig. 8(d), depicts the situation in which $\Lambda(r) = r$ and (b_r) is as shown in Figs. 6(c) and 7(c); i.e., the pseudo-statistical variations in the modal overlap parameter (b_r) entertains values that largely exceed unity. Fig. 8(d) shows that at and in the vicinity of the resonance frequencies of those satellite oscillators to which (b_r) are assigned values that approach and exceed unity; e.g., as depicted in Figs. 6(c) and 7(c), the fluctuations are largely subdued; e.g., see Fig. 8(c) and contrast it with Fig. 8(d) [19]. The fifth and sixth figures in the set, e.g., Figs. 8(e) and (f), depict the combined situation in which $\Lambda(r)$ and (b_r) are as shown in Figs. 8(a) and (b) and in Figs. 8(a) and (c), respectively. When variations in both parameters are combined; e.g., as depicted in Figs. 8(e) and (f), both characteristics can be identified in the levels of the induced loss factor $\eta_I(y)$; e.g., see Figs. 8(e) and (f), and contrast them, respectively, with the pair Figs. 8(b) and (c), and then with the pair Figs. 8(b) and (d) [1]. That fluctuations do occur in the response character of the complex, when parameters that describe it undergo changes, is not

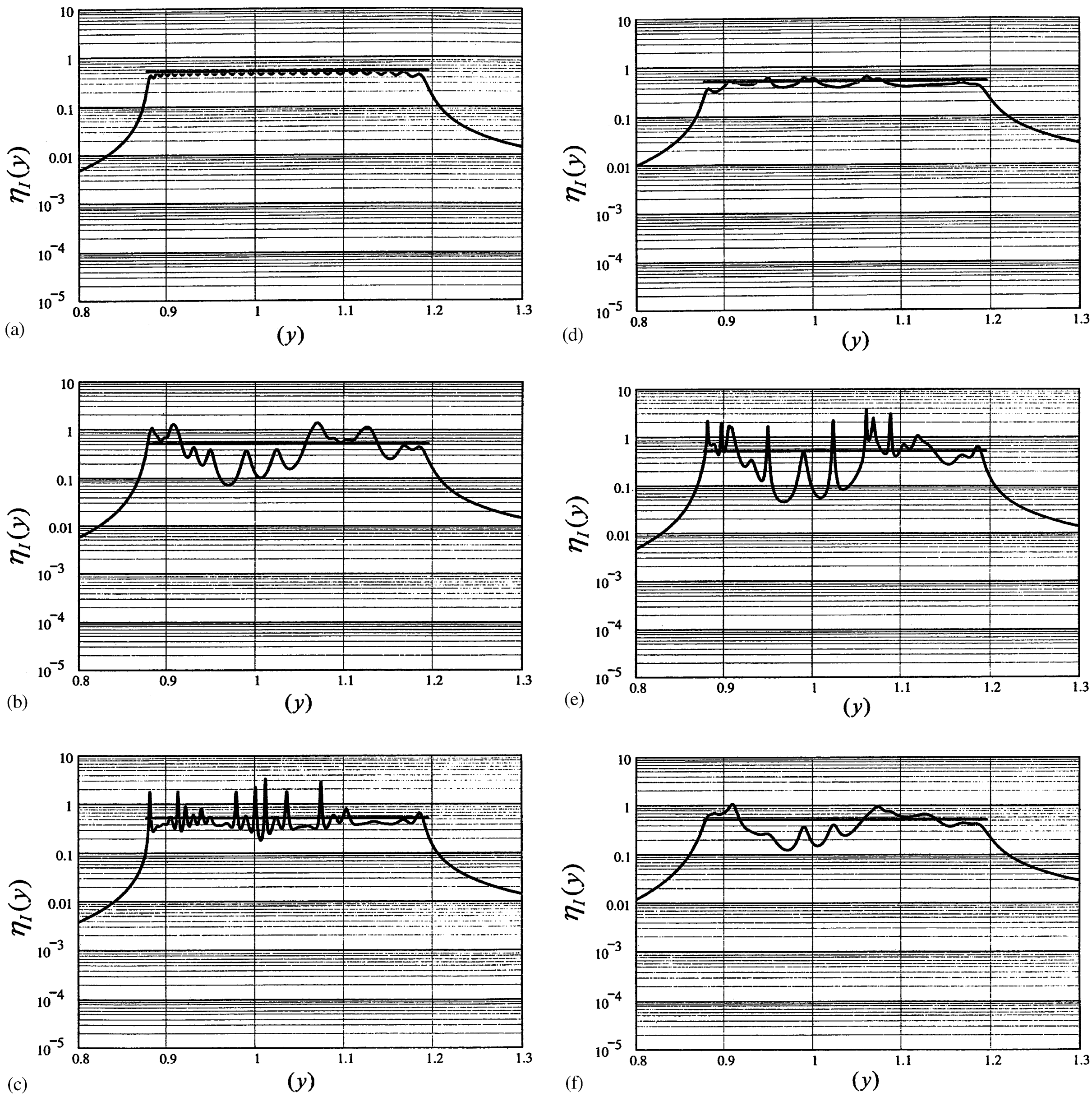

Fig. 8. Induced loss factor $\eta_I(y)$, as a function of (y), for a strong coupling defined by $\alpha_c = 1.0[\alpha = 0.0]$, $\bar{m}_c = g = 0$; implementing a number of pseudo-statistical variations ($R=27$ and $\left(M_s/M\right)=0.1$). First order approximation (FOA) is superimposed. (a) $\Lambda(r)=r$ and $b_r=1$, a base case. (b) $\Lambda(r)$ as given in Fig. 6(a) and $b_r=1$. (c) $\Lambda(r)=r$ and (b_r) as is given in Fig. 6(b). (d) $\Lambda(r)=r$ and (b_r) as given in Fig. 6(c). (e) $\Lambda(r)$ as given in Fig. 6(a) and (b_r) as is given in Fig. 6(b). (f) $\Lambda(r)$ as given in Fig. 6(a) and (b_r) as is given in Fig. 6(c).

surprising. The purpose of this section is, however, to argue that fluctuations of the kind typified in Fig. 8 need be kept in mind when presented with data of practical dynamic systems. The results presented in this figure may help avoid either reading too much into such fluctuations or too little.

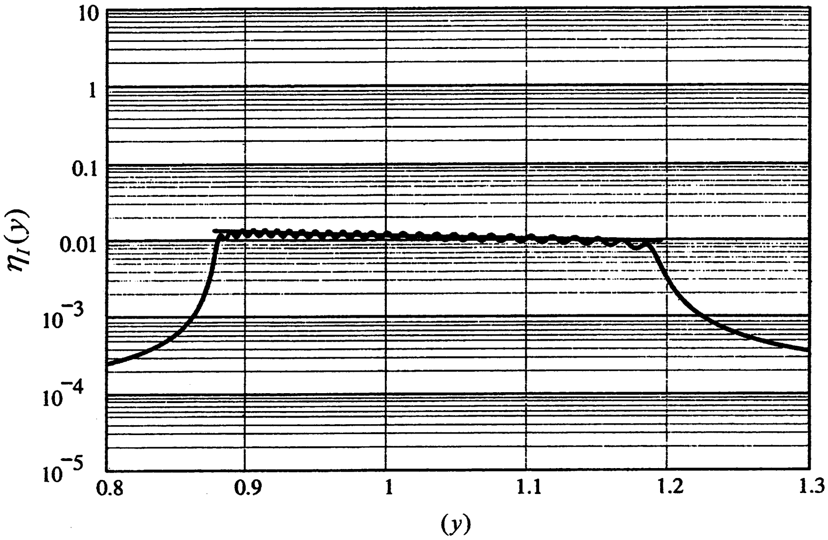

Fig. 9. As in Fig. 8(a), except that the coupling is in the form of a mix of stiffness and gyroscopic elements: $\alpha_c = 0.1[\alpha = 0.9]$, $g = 0.11$, $\bar{m}_c = 0$. The coupling strength here is moderate and the modal overlap parameter (b) is equal to unity.

References

[1] G. Maidanik, K.J. Becker, Induced changes in the impedance of a master oscillator due to coupling to a set of satellite oscillators: linear analysis, NSWCCD-70-TR-2001/098, 2001.

[2] M.J. Brennan, Wideband vibration neutralizer, Noise Control Engineering Journal 45 (1997) 201–207.

[3] G. Maidanik, K.J. Becker, Noise Control of a master harmonic oscillator coupled to a set of satellite harmonic oscillators, Journal of the Acoustical Society of America 104 (1998) 2628–2637;
G. Maidanik, K.J. Becker, Characteristics of multiple-sprung mass for wideband noise control, Journal of the Acoustical Society of America 106 (1999) 3119–3127.

[4] G. Maidanik, Power dissipation in a sprung mass attached to a master structure, Journal of the Acoustical Society of America 98 (1995) 3527–3533.

[5] A. Pierce, W.V. Sparrow, D.A. Russell, Fundamental structural-acoustic idealizations for structures with fuzzy internals, Journal of Acoustics and Vibration 117 (1995) 339–348.

[6] M. Strasberg, D. Feit, Vibration damping of large structures by attached small resonant structures, Journal of the Acoustical Society of America 99 (1996) 335–344.

[7] R.L. Weaver, Mean and mean-square responses of a prototypical master/fuzzy structure, Journal of the Acoustical Society of America 99 (1996) 2528–2529.

[8] R.J. Nagem, I. Veljkovic, G. Sandri, Vibration damping by a continuous distribution of undamped oscillators, Journal of Sound and Vibration 207 (1997) 429–434.

[9] T.L. Smith, K. Rao, I. Dyer, Attenuation of plate flexural waves by a layer of dynamic absorbers, Noise Control Engineering Journal 26 (1986) 56–60.

[10] G. Maidanik, J. Dickey, Singly and regularly ribbed panels, Journal of Sound and Vibration 123 (1988) 309–314.

[11] Yu.A. Kobelev, Absorption of sound waves in a thin layer, Soviet Physics Acoustics 33 (1987) 295–296.

[12] G. Maidanik, Induced damping by a nearly continuous distribution of nearly undamped oscillators: linear analysis, Journal of Sound and Vibration 240 (2001) 717–731.

[13] R.H. Lyon, Statistical Energy Analysis of Dynamical Systems: theory and applications, MIT, Cambridge, 1975;
R. H. Lyon, R. G. Dejung, Theory and Application of Statistical Energy Analysis, Butterworth-Heinemann, Boston, 1995.

[14] R.H. Lyon, G. Maidanik, Power flow between linearly coupled oscillators, Journal of the Acoustical Society of America 34 (1962) 623–639.
[15] G. Maidanik, K.J. Becker, Dependence of the induced loss factor on the coupling forms and couplings strengths: energy analysis, Journal of Sound and Vibration 266 (2003) 33–48, this issue.
[16] E. Skudrzyk, The mean-value method of predicting the dynamic response of complex vibrators, Journal of the Acoustical Society of America 67 (1980) 1105–1135.
[17] M.R. Spiegel, Complex Variables with an Introduction to Conformal Mapping and its Applications, Schaum's Outline Series, McGraw-Hill Book Company, New York, 1964.
[18] G. Maidanik, K.J. Becker, Modal densities of simple dynamic systems, Journal of the Acoustical Society of America 102 (1997) 3130A.
[19] M. Strasberg, When is a "fuzzy" not a fuzzy (Continued)?, Journal of the Acoustical Society of America 107 (2000) 2885A.